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Unrealistic Responses to Realistic Problems With Missing Information: What are Important Barriers?

Abstract

It is a well-documented finding that students tend to neglect their real-world knowledge when solving word problems, even when realistic assumptions are needed. Although studies have successfully shown the extent to which students tend to provide unrealistic responses, the question of where this tendency comes from has yet to be answered. We focused on two major steps needed to solve realistic word problems: noticing missing information and making realistic assumptions. We conducted two studies with fifth graders (Study 1, *N*=108; Study 2, *N*=60) in which we compared students' (un-)realistic responses to problems that differed in how obvious the missing information was. Study 1 fostered only students' ability to make assumptions. Study 2 fostered this ability plus the ability to notice missing information. The results indicate that, if the missing information is not obvious, students' failure to notice it seems to be what prevents them from arriving at a realistic solution.

Keywords: Assumptions, multiple solutions, P-items, realistic reasoning

Unrealistic Responses to Realistic Problems With Missing Information: What are Important Barriers?

One of the central goals of mathematics education is to teach students to identify and understand the connection between the real world and mathematics (Blum, 2007; Niss, Blum, & Galbraith, 2007). The most common way to bring the real world into classrooms is by presenting word problems (Verschaffel, Greer, & de Corte, 2000). Word problems include a great variety of tasks that can be categorized according to different criteria. They are often characterized by their mathematical content, such as arithmetic word problems (Kintsch & Greeno, 1985; Nortvedt, 2011; Riley, Greeno, & Heller, 1983), algebraic word problems (Hinsley, Hayes, & Simon, 1977; Reed, Stebick, Comey, & Carroll, 2012), or statistical word problems (Quilici & Mayer, 1996). Another criterion refers to their relation to reality (Krawitz & Schukajlow, 2017), ranging from “dressed-up” word problems to complex modelling tasks (Niss et al., 2007). Prior research has demonstrated that students often struggle when a problem’s solution requires them to consider realistic contextual aspects that are described in the word problem (Greer, 1993; Verschaffel, De Corte, & Lasure, 1994; 2000; Yoshida, Verschaffel, & De Corte, 1997). One specific feature of realistic word problems is that they often do not contain all of the information required to obtain a correct solution. Such problems, which we refer to here as problems with missing information, foster skills such as estimation skills, which are considered to be important for students’ actual and later lives (Ärlebäck, 2009; Peter-Koop, 2009; Sriraman & Lesh, 2006).

In the present paper, we focus on word problems that are presented from a realistic perspective and are missing information. While there are several steps in the problem solving cycle that seem important to solve such problems in a realistic manner, we specifically focus on two of these steps: one is that students have to notice that the word problem is missing information; the other is that they have to make assumptions about this missing information by identifying and replacing the missing elements. Our aim is to clarify whether a failure to notice missing information is a major reason for why students neglect real world issues while solving such problems.

Theoretical Background

Students' Tendency to Neglect Real-World Issues While Solving Word Problems

Regardless of the recent educational debate that has emphasized the importance of realistic word problems for students' current and future lives and has suggested the use of such problems in the classroom (Blum, 2007; Niss et al., 2007), the word problems found in textbooks are typically well-structured as all the necessary information is given. This is because such problems are explicitly designed to provide a way for students to practice the mathematical procedures they have just learned (Nesher, 1980). However, such word problems tend to be "artificial" because the only purpose of the real-world context is to "dress up" a mathematical task (Krawitz & Schukajlow, 2017). Thus, several authors have argued that this frequent use of artificial word problems and the manner in which such tasks are applied in teaching-learning situations have caused students to develop restricted beliefs about word problems. Specifically, such beliefs are that all of the relevant information is given, every problem has a single precise numerical answer, and all of the given numbers are relevant for the solution (Jiménez & Verschaffel, 2013; Reusser & Stebler, 1997). Further, these beliefs are considered to promote superficial solution strategies and hinder realistic considerations such as noticing when there is missing information and making realistic assumptions (Reusser & Stebler, 1997; Verschaffel, Greer, Van Dooren, & Mukhopadhyay, 2009).

To support these claims, several studies were conducted in which students were confronted with problematic items (P-items) consisting of word problems that look at a first glance like traditional word problems but require realistic thinking (Dewolf, Van Dooren, Cimen, & Verschaffel, 2013; Greer, 1993; Reusser & Stebler, 1997; Verschaffel et al., 1994; 2000; Yoshida et al., 1997). The following problem is an example for a P-item: “Mr. Meier wants to have a rope long enough to stretch between two poles that are spaced 12 m apart, but he has only pieces of rope that are 2 m long. How many of these pieces would he need to tie together to stretch between the poles?" (adapted from Greer, 1993). An unrealistic answer would be ‘six pieces’, because it ignores the situational constraint that additional rope is needed to tie the pieces of rope together and to the poles, whereas a realistic response would take into account that additional rope is needed. In this article, we focus on P-items that require realistic considerations because some information is missing. We chose such items because P-items are particularly designed to investigate students’ ability to include realistic considerations in their reasoning. Further, we decided to focus only on missing information problems for reasons of better comparability and because dealing with missing information is crucial for everyday life (Ärlebäck, 2009; Jonassen, 2000; Peter-Koop, 2009; Sriraman & Lesh, 2006).

Research has provided numerous theoretical models on the solution processes for realistic word problems and has described the different steps that are needed to solve these problems (Blum & Leiß, 2007; Galbraith & Stillman, 2006; Staub & Reusser, 1995). Any of the steps might be demanding for students, and any step can be a potential barrier, but the different steps have different levels of importance depending on the type of word problem. With respect to solving realistic word problems with missing information, we focus on the activities of simplifying and structuring from prior theoretical models (Blum & Leiß, 2007).

We further focus on the following steps as we argue that they are essential for arriving at an appropriate solution:

(1) Noticing missing information

(2) Making realistic assumptions, including identifying and supplementing the missing quantities

To solve realistic word problems with missing information, these points can be considered essential as exemplified by the following analysis of the "Rope" Problem. To solve this problem, it is essential to take into account the reality of the situation as presented in the problem statement, to notice that information is missing from a real-world perspective, and to make assumptions about the missing information. In the "Rope" problem, this would be the length of rope that is needed to tie two pieces of rope together and to tie the ropes around the poles and to estimate the length needed to accomplish these goals. At first glance, it appears to be sufficient to take the given numerical values and to apply arithmetic operations to them in a straightforward way. However, applying straightforward arithmetic operations would ignore the realistic context and lead to an unrealistic answer (in the "Rope" problem presented above: 6 pieces).

Studies investigating P-items have impressively demonstrated students' strong tendency to ignore realistic issues while solving P-items that are missing information (Greer, 1993; Verschaffel et al., 1994). The findings have been replicated several times in various countries around the world, thus demonstrating that students tend to fail to consider the realistic aspects of the problems (Dewolf et al., 2013; Reusser & Stebler, 1997; Yoshida et al., 1997).

Noticing Missing Information

In several studies, researchers have attempted to increase the number of realistic responses to P-items by alerting the students (Dewolf et al., 2013; Reusser & Stebler, 1997; Yoshida et al., 1997). Alerting students was expected to make students pay more attention while solving the problems so that they would notice missing information. However, findings have shown that students' tendency to provide unrealistic solutions is persistent and difficult to change by offering general warnings (Dewolf et al., 2013; Reusser & Stebler, 1997; Yoshida et al., 1997). For example, at the tops of the test sheets, Reusser and Stebler (1997) printed a general instructional signal that read: "Be careful! Some of the following problems aren't as easy as they look. There are, in fact, some problems in the booklet where it is very questionable if they are solvable at all." (Reusser & Stebler, 1997, P. 319).

However, there is also empirical evidence for a positive impact of instructional signals, which were directly embedded into the formulations of the problems (Reusser & Stebler, 1997; Weyns, Van Dooren, Dewolf, & Verschaffel, 2016). For example, for the "Rope" item, the problem-specific instructional signal was: "Think about it carefully before you answer!" (Reusser & Stebler, 1997, S. 319). For the "Rope" item, 32.5% of the students who received the problem-specific instructional signal gave a realistic response in comparison with 22.2% in the condition with a general instructional signal and 21.7% in the condition without an instructional signal. One reason for these effects might be that students in the problem-specific instructional signal condition noticed that some information was missing from the "Rope" item. However, we could not find any studies that investigated whether specifically a failure to notice missing information was a major barrier to providing realistic responses for P-items with missing information.

Another approach that can be applied to address students' ability to notice missing information is prompting students to find two different solutions for each item. The multiple solution approach was found to have positive effects on solving intra-mathematical problems (Rittle-Johnson & Star, 2007) and in particular on realistic word problems with missing information (Schukajlow & Krug, 2013a). The existence of multiple solutions for the same problem is a typical feature of problems with missing information because different assumptions about the missing values lead to different outcomes, and thus multiple solutions are possible (for a comparison of different categories of multiple solutions see Schukajlow & Krug, 2014; Tsamir, Tirosh, Tabach, & Levenson, 2010). Particularly important for the present study is the finding that prompting students to find multiple solutions positively affects metacognitive activities such as planning and monitoring the solution process (Schukajlow & Krug, 2013a). Metacognitively reflecting on the solution process and the results play important roles in students' ability to evaluate the mental model of the situation (Stillman, 2011) and might therefore stimulate students to notice missing information. The request for and thus the existence of a second solution means that the problem differs from the well-structured problems students are used to. Thereby, the request for two solutions is similar to approaches in which specific instructional signals are issued because the request for a second solution cannot be overlooked. However, to the best of our knowledge, no studies have explored the impact of prompting students to find multiple solutions on the occurrence of students' realistic considerations while solving P-items.

Noticing Missing Information and Making Assumptions about Missing Information

Approaches in which both of the solution steps that are the focus of the current paper (i.e., noticing and making assumptions about missing information) are addressed to solve P-items can be found in long-term interventions with a focus on mathematical modelling (Verschaffel & De Corte, 1997; Verschaffel, De Corte, & Lasure, 1999). More specifically, these long-term interventions have included (a) metacognitive reflection through which students' ability to notice the problematic aspects of the problems was addressed and (b) exercises to foster students' ability to make assumptions. Furthermore, interventions also implemented more global issues such as strategic mentoring and variations in teaching methods to promote learner activities. A positive impact of these long-term interventions was found, indicating that realistic responses can be increased by fostering both solution steps that is, noticing and making assumptions. However, as a lot of different teaching elements (group work, teaching interventions, and others) have been addressed in these studies, definitive conclusions cannot be reached regarding which of the various steps involved in solving P-items is responsible for students' unrealistic responses. Thus, research has not yet determined which steps are most important to address in interventions that are designed to promote realistic considerations. Our approach focuses on two important steps, noticing missing information and making realistic assumptions, from which we do not know whether trouble in one or the other leads to a lack in students' realistic responses.

The Present Studies: Contrasting P-items with Items in Which Missing Information is Obvious

As mentioned above, P-items have the feature that they look similar to well-structured word problems. This feature might lead to difficulties with the first of the selected steps: noticing missing information. Failure on this step means students will have no chance of finding an appropriate solution. Thus, we expected to find that a failure to notice missing information is the crucial barrier to finding realistic solutions to P-items. If this is the case, students should be able to find realistic solutions to word problems that are obviously different from well-structured word problems, even if such problems require students to make assumptions (Ärlebäck, 2009; Peter-Koop, 2009). For example, if a problem does not contain any numbers but still requires a numerical answer, missing information cannot be easily overlooked, and a superficial solution is not possible. Take for example the following problem: "How many centimeters of toothpaste are used in a month?" (adapted from Bülow et al., 2013). Given that the problem is asking for a numerical answer when there is no numerical information presented in the problem whatsoever, it is rather obvious that more information is needed to find a solution. In the following, we address these items as problems in which missing information is obvious (O-items). In contrast to P-items, it can be expected that noticing that information is missing will be a lot easier for O-items. However, making assumptions can be required by both P-items and O-items, as making realistic assumptions requires the cognitively demanding activity of setting up a proper mental model of the situation (Blum, 2015). Trying to figure out and complement important information without losing sight of the underlying connections can be difficult depending on the complexity of the problem at hand (Krutetskii, 1976).

To clarify whether differences in the realistic responses to P-items and O-items rather come from noticing that there is missing information or from making assumptions on this information, we created treatments for addressing both possible barriers while students solved these types of problems. As it is known that students are usually not familiar with the development of multiple solutions, asking them to provide two solutions when they have not practiced doing so will be likely to overstress them (Silver, Ghousseini, Gosen, Charalambous, & Font Strawhun, 2005). Considering this, instead of addressing only the activity of noticing missing information in a single treatment, we decided to use an indirect approach in which two treatments were combined: The first treatment (Study 1) was designed to foster students’ ability to make assumptions, whereas the second (Study 2) was designed to foster both the ability to make assumptions and the ability to notice missing information. Combining the results of the two studies allowed us to conclude whether a failure to notice missing information was the central barrier to solving P-items with missing information.

Study 1: Effects of Fostering Students to Make Assumptions on Realistic Responses

Research Questions and Hypotheses

The considerations presented in the theoretical framework led to the following research questions:

(1) Will students give more realistic responses to problems in which missing information is obvious (O-items) than to problems in which missing information is not obvious (P-items)?

(2) Does fostering students' ability to make assumptions influence their ability to provide realistic responses to P-items and O-items? Will such practicing and discussing have a stronger positive impact on O-items than on P-items?

In the previous section, we considered the first step in solving P-items (i.e., noticing missing information) to be an essential part of the solution process, whereas this is not necessarily the case for O-items because it is more obvious that information is missing in O-items. We therefore expected that P-items would less often be solved in a realistic manner than O-items.

Further, we expected positive effects of practicing and discussing an example solution on realistic responses to O-items but not to P-items because the activities presented in a brief session in which a sample task was practiced and discussed are supposed to foster the second solution step: making assumptions about the missing information. For P-items, learners might not realize that making assumptions is essential because they do not notice missing information and hence the necessity to make assumptions. Thus, students might fail to master the first step, at least in part, and they might therefore stick with their unrealistic responses when solving P-items.

Method

Sample and design. The present study involved 108 fifth graders (52% female, mean age = 10.8 years) from four high-track classes (German gymnasium) from two different schools. One class from each school was randomly assigned to the experimental group (EG) and another class to the control group (CG). Both groups included a similar number of students from each school (Students in the EG: 26 from School 1, 26 from School 2; CG: 27 from School 1, 29 from School 2). There were no significant differences between the groups with respect to students' mathematical ability as measured with a standardized mathematical computation test (DEMAT 5+ from Götz, Lingel & Schneider, 2013) (EG: $M = 6.135$, $SD = 3.087$; CG: $M = 6.357$, $SD = 3.006$; $t(106) = -.378$, $p = .71$). Furthermore, the groups did not differ in their experience with problems with missing information (EG: $M = 2.615$, $SD = 0.591$; CG: $M = 2.705$, $SD = 0.693$; $t(106) = -.724$, $p = .471$) measured with a questionnaire with four items on a four-point Likert scale (1 = not at all true, 4 = completely true). The four items were adapted from Schukajlow and Krug (2013b). A sample task is: "In mathematics, we often solve problems where missing information can be supplemented." The reliability value (Cronbach's α) of the scale was .678.

Treatment. Both groups received brief general instructions for solving problems with missing information (similar to the instruction used in the study by Dewolf et al., 2013) in order to prevent students from not being able to complete the test just because they were not familiar with these types of problems. A trained test supervisor orally presented the instructions in a collective classroom discussion:

> "You will see that the following problems differ from the problems you are usually given because they are missing information. These problems are nevertheless solvable. To solve these problems, you have to make assumptions about the missing information."

In addition, the students in the EG briefly practiced and discussed a problem that was missing information. The practicing consisted of the following sample problem “Jellybeans,” which the students were prompted to solve in the classroom:

> “Some friends meet to play. They have 20 small bags of jellybeans. How many bags does each of them get?”

After five minutes, one of the students presented his/her solution and explained how it was found. Then, as a class, students discussed how an assumption about the number of friends was essential to solve this problem. The problem “Jellybeans” was chosen for the present study because it is not a P-item or an O-item. This problem includes features of each of the problem types. Only a single number (i.e., 20) is given. To make a calculation, a second number would be required (in this case, the number of friends) as well as the assumption that every friend gets the same number of jellybeans.

Measures. The test included the abovementioned P-items and O-items. P-items appear to be solvable by applying standard operations in a straightforward way, and this makes it more difficult to recognize that it is necessary to make assumptions. For O-items, on the other hand, the fact that information is missing cannot be overlooked because the problem statement does not contain numerical information at all (see Table 1).

INSERT TABLE 1 ABOUT HERE

Students’ solutions to the items were scored as “realistic” if the solution included realistic assumptions. The assumptions did not have to be explicitly stated, and arithmetic errors were tolerated. For the P-items, students had to mention or at least implicitly show in their calculations that additional rope was required to tie the pieces together and/or to tie the ropes to the poles (“Rope”) and that more ribbon was needed to tie a bow (“Present”). For the O-items, students had to make realistic assumptions about the length of toothpaste per brush, how often one brushes each day, and the number of days in one month (“Toothpaste”). They also had to guess the number of guests and the number of chocolate marshmallows per pack ("Birthday" item). In addition, responses to the "Birthday" item were still coded as realistic even if they did not state that Max and his mother probably also wanted to eat chocolate marshmallows as long students stated that there had to be enough for the guests. This coding was used because the issue that Max has to be considered is not easy to recognize, and we wanted to eliminate the "problematic" part of the item to ensure that it could be considered an O-item. A second rater independently scored 19.4% of the data with regard to the realistic assumptions of the solutions. The resulting agreement with the first rater was satisfactory (κ = .691).

Analysis steps. To answer the first research question (Differences in Solving P-items and O-items) we considered the solutions of the CG to examine the differences between realistic responses to the P-items and O-items. Hence, we compared each of the P-items with each of the O-items. We used a logistic regression with the occurrence of realistic responses as the dependent variable and the problem type as the independent variable to account for the four combinations of P-items and O-items. The second research question (Impact of Fostering Students' Ability to Make Assumptions on Realistic Responses) was examined by computing a logistic regression in which the occurrence of realistic responses was the dependent variable and the short-term intervention was the independent variable.

Results

Descriptive Results. Only 23.8% of the responses to all problems were scored as realistic. The percentages and total numbers of realistic responses to both types of problems are presented in Table 2.

INSERT TABLE 2 ABOUT HERE

Differences in Solving P-items and O-items. Table 3 shows the results of the logistic regression analysis for answering the first research question.

INSERT TABLE 3 ABOUT HERE

The results showed that the odds of providing a realistic response to the O-item “Toothpaste” were significantly higher than for each of the P-items (6 and 30 times greater odds). For the comparison of the “Birthday” problem with the P-items, the results were indeed in the expected direction (2 and 9 times greater odds) but were not significant. Therefore, our expectation that both O-items would be statistically significantly more often solved in a realistic manner was partially confirmed.

Impact of Fostering Students’ Ability to Make Assumptions on Realistic Responses. Table 3 includes the results of the logistic regression analysis for answering the second research question. The results revealed that the impact of practicing and discussing differed with respect to the problem type. In line with our expectations, we found no effects of practicing on the realistic responses to the P-items. In contrast to this, as hypothesized, for the two O-items (“Toothpaste” and “Birthday”), students in the EG had significantly higher odds (3 and 8 times greater) of giving a realistic response than the students in the CG. Practicing and discussing were successful for O-items but not for P-items. Thus, our expectation of a positive impact of practicing on O-items in contrast to P-items was confirmed.

Discussion

The findings supported our expectations, and significantly more realistic responses were found for the O-items than for the P-items. This finding can be considered as a first indication that recognizing missing information is a key difficulty in solving P-items because this is the central feature by which P-items differ from O-items. At first glance, the P-items are similar to the well-structured word problems found in textbooks, and they seem to be solvable by applying standard operations. This might impede the search for realistic solutions: When solving P-items, students might focus on the given numbers in combination with the suggested standard operations and therefore fail to create a sufficient mental model of the problem (Krutetskii, 1976; Reusser & Stebler, 1997; Stillman & Galbraith, 1998).

Taking into account the interaction between the condition and the problem type, a significant effect of practicing and discussing on both O-items was found, whereas this was not the case for the P-items. We could therefore conclude that fostering students' ability to make assumptions helps them solve O-items in a realistic manner, but more effort is needed for P-items. It seems that unrealistic considerations while solving P-items can be explained by students' trouble noticing missing information in such problems.

Study 2: Multiple Solutions to Problems with Missing Information

The second study used a multiple solution approach to additionally address students' ability to notice missing information.

Research Question

We investigated the following research question: Does fostering students' ability to notice missing information and make assumptions have a positive impact on the occurrence of realistic considerations? If yes, is the impact on P-items greater in comparison with O-items?

The instructions included practicing and discussing an example that was similar to the treatment used in Study 1. The activity of making assumptions was therefore also addressed in the treatment used in Study 2. Hence, we expected a positive effect on O-items. In addition, we expected that prompting students to provide two solutions would stimulate them to notice missing information and therefore lead to more realistic responses to P-items. First, students' ability to notice missing information might be fostered by stimulating their metacognitive activities (Schukajlow & Krug, 2013a). And second, the request for a second solution suggest that the problem is not well-defined. Hence, prompting students to provide two solutions probably works as a concrete warning, which was shown to be helpful for solving P-items (Reusser & Stebler, 1997).

Method

Sample and Design. Participants were 60 fifth graders (55% female, mean age = 10.6 years) from four high-track classes (German gymnasium) from two different schools. They did not participate in Study 1. We used an extended version of the general instructions for problems with missing information from Study 1. In addition to presenting the instructions from Study 1—which stated that the problems could be solved by making assumptions—students were prompted to find two different solutions to the problems in order to stimulate their ability to notice missing information: "You have to find two different solutions to each of the following problems. This means you need to find two different results and not two different calculations that lead to the same result."

After the instructions, the students were asked orally to find one solution to the "Jellybeans" problem from Study 1. The students presented their different solutions, and the class discussed why there was more than one solution. The conclusion of the discussion was that different assumptions lead to different solutions, but both solutions are right. After this intervention, a test including the same four missing-information problems from Study 1 was administered with the difference that students were prompted to produce two solutions to each problem.

The coding of students' first and second solutions was the same as in Study 1. Interrater reliability was computed for 16.7% of the cases, with very good agreement (κ = .826).

Analysis Steps. We used an explorative approach in which we first, compared the rates of realistic first and second responses of the second study to the rates of realistic responses of the first study, second, compared the rates of realistic first responses to the rates of realistic second responses for each problem of the second study, and third, analyzed students' solutions and categorized them in groups to reveal common mistakes.

Results

The research question asked whether instructing students to find two different solutions would impact their realistic considerations. A total of 37.1% of first responses and 41.3% of second responses were scored as realistic. Both rates of realistic responses were higher than the overall rate of realistic responses (23.8%) and even higher than the rate of realistic responses found in the EG (33.2%) in the first study.

It is remarkable that for the P-item "Rope"—but not for the "Present" item—students more often gave a realistic second than a realistic first solution (see Table 4), whereas the O-items showed similar percentages of realistic responses for the first and second solutions.

INSERT TABLE 4 ABOUT HERE

In particular, we found for the "Rope" item that the large majority of first solutions (88.3%) involved the direct application of operations to the given numbers (12/2 = 6), whereas for the second solutions, only 22.3% of the students gave this answer. Instead, more realistic considerations but also more missing answers were found in the second solutions.

However, for the second P-item "Present", the numbers of first and second solutions that were realistic were the same. An analysis of students' mistakes showed that they often struggled to set up a mathematical expression. Just multiplying the given lengths was a very common mistake (20.0% of first and 13.3% of second solutions). Also, frequent answers were 50 cm and 40 cm, which also often appeared in combination as the first and second solutions (21.7%). These outcomes result from tying the present only lengthwise and choosing for this the length or the width of the rectangle, neglecting the need for additional ribbon for a bow. Table 4 also presents the relations between the first and second solutions, in other words, how often an unrealistic first solution was followed by a realistic second solution and vice versa.

For the O-items as well as for the “Present” item, there were only a few cases in which students’ first and second answers differed concerning their connection to reality. It is interesting that this was not the case for the “Rope” item. For this item, 25.0% of the students gave a realistic second response after they gave an unrealistic first solution. This result is even more striking when taking into consideration the fact that the vast majority of realistic responses for this item were given after an unrealistic first solution.

Discussion

Study 2 was conducted to investigate the impact of promoting students’ abilities to notice missing information and to make assumptions when solving P-items by asking them to provide multiple solutions and having them practice a sample problem. Using the results of Study 1 as a baseline, for each of the problems, the number of students who found at least one realistic solution was higher in Study 2 than in Study 1. This was still the case when only the first or even only the second solution was considered. Hence, addressing the first solution step (i.e., noticing missing information) seams to lead to more realistic responses to P-items and O-items.

The comparison of the first and second solutions in Study 2 showed no differences for the “Tooth,” “Birthday,” and “Present” items, in contrast to the “Rope” item, where a much larger number of realistic second than realistic first solutions was found. Thus, prompting students to find two solutions seems to lead to more realistic responses particularly for problems where only one straightforward application of standard mathematical procedures is possible, as it is the case for the “Rope” problem. After a first solution is produced, such P-items might become similar to O-items because all of the given numbers have already been used. Regarding the second P-item (“Present” problem), the number of realistic considerations did not increase after students were asked to develop two solutions. Thus, students seem to include reality in their considerations only if they do not have any other way to solve the problem. This finding supports prior findings that showed that the tendency to neglect reality while solving realistic word problems is very persistent (Dewolf et al., 2013; Verschaffel et al., 2000; Yoshida et al., 1997).

Summary and Concluding Discussion

Problems with missing information are an important part of mathematics education as well as real life (Blum, 2015; Jonassen, 2000; Maaß, 2010). Solving problems with missing information requires skills and solution strategies that are different from the ones required to solve well-structured problems (Jonassen, 2000; Krutetskii, 1976; Spiro, Feltovich, Jacobson, & Coulson, 1991). However, we do not know much about the issues that occur while solving problems with missing information. In the present studies, we investigated students’ (un-)realistic responses to problems with missing information, which we distinguished by the extent to which the missing information was obvious: P-items and O-items. We claim that P-items require an important step in the solution process, namely, the step in which the problem solver must notice that information is missing, whereas the necessity to make assumptions can hardly be overlooked in O-items because no numbers are presented for such items even though the problem requires a number as an answer.

Summarizing the findings of both studies, we found more realistic responses to O-items than to P-items, supporting our hypothesis that it is more difficult for students to provide realistic solutions to P-items than to O-items. Further, we found that fostering students’ ability to make assumptions did not increase their realistic responses to P-items, whereas it did increase their realistic responses to O-items, thus supporting our hypothesis that deficits in making assumptions are not the most prominent reason that students fail to solve P-items. However, also fostering students' ability to notice missing information led to more realistic responses to the P-items as well, indicating that noticing missing information is a major barrier to solving P-items.

Still, it is important to stress that, similar to prior studies (Greer, 1993; Verschaffel et al., 1994; Yoshida et al., 1997), the number of realistic responses to P-items was low, even after both solution steps were fostered. The finding of 26.7% realistic responses given as a second solution for the "Rope" problem nevertheless indicates that the multiple solutions approach, which has been shown to be beneficial for performance in solving intramathematical problems (Rittle-Johnson & Star, 2007) or in solving word problems with missing information (Schukajlow & Krug, 2013a; Schukajlow, Krug, & Rakoczy, 2015), also seems to be promising for increasing realistic responses and should be addressed in future research.

Moreover, our results extend the research on word problems with missing information (Ärlebäck, 2009; Peter-Koop, 2009) as we demonstrated that students' difficulties in solving these problems greatly depend on the solution steps required to solve problems with missing information.

Strengths and Limitations

In the present research, we combined two studies to investigate students' tendency to neglect real-world issues. One limitation of Study 1 concerns the choice of the sample problem. We decided to use a missing information problem that had one numerical data point and thus differed from the P-items and also from the O-items that were used in the study. However, it is possible that the example problem was more similar to the O-items than to the P-items. As treatment effects tend to be stronger when the example problems presented to the treatment group are closely related to the target content (Hattie, Biggs, & Purdie, 1996), the example problem may have increased the impact of the treatment for the O-items but not for the P-items. Although numerical information was given in the example problem (i.e., the number 20), it was obvious that mathematical operations could not be applied to just that single number. Just picking the 20 itself would also be an unrealistic solution, but this violates students' belief that it is always necessary to do calculations (Jiménez & Verschaffel, 2013). Further, because of the time limit, we used only two P-items and two O-items. We acknowledge that the small number of items may cause a bias. If one of our items is perceived differently by subgroups within the experimental and control groups, this effect cannot be absorbed by other items. The rationale behind this limitation was that we used the test time to address students' arithmetical performance, which was crucial for ensuring that the experimental and control conditions would be comparable. Expanding the test time might result in a decrease in motivation, which is important for the validity of the results. Future studies could increase the number of items in order to improve the reliability and generalizability of the results. However, a larger number of items could also lead to learning effects, which could occur during the test itself. Another limitation concerns randomization at the class level. We compared the groups regarding their mathematical ability and their experience with problems with missing information. However, we did not use other important variables like the socio-economic status for a comparison. Future studies could use a randomization on the student level to avoid class effects.

Implications for Research and Practice

Our approach combined two lines of research, research on realistic reasoning (Dewolf et al., 2013; Reusser & Stebler, 1997; Verschaffel et al., 1994) and research on missing information (Ärlebäck, 2009; Peter-Koop, 2009; Sriraman & Lesh, 2006). Combining two research perspectives, allowed a closer look on potential barriers that occur while solving problems with missing information in a realistic manner. The findings indicate that differences among missing information problems regarding how obvious missing information is, have to be taken into account in future studies. The link between multiple solutions and realistic considerations while solving realistic word problems had not been explored previously, and so our aim was to prepare the ground for further research. One future issue might be the investigation of the role of metacognitive activities for realistic reasoning, as metacognitive activities increased after solving problems with multiple solutions (Schukajlow & Krug, 2013a). To address the importance of realistic considerations, asking students to compare, link, and contrast realistic and unrealistic solutions can be expected to change students' beliefs about realistic word problems that were found to be important for realistic reasoning (Jiménez & Verschaffel, 2013; Reusser & Stebler, 1997). From a practical perspective, the findings from practicing and discussing a problem with missing information demonstrated that for some problems with missing information, it is beneficial to solve and discuss a sample problem in class (O-items); however, more effort is required for problems in which it is difficult to notice missing information (P-items). For P-items, the results of Study 2 seem to be promising. Prompting students to find multiple solutions seems to make it easier for students to come up with realistic solutions at least for some P-items for which only a single unrealistic solution is possible.

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Tables

Table 1

The P-items and O-items Given on the Test

P-items	Rope	Mr. Meier wants to have a rope long enough to stretch between two poles that are spaced 12 m apart, but he has only pieces of rope that are 2 m long. How many of these pieces would he need to tie together to stretch between the poles? (adapted from Greer, 1993)
	Present	Sina giftwraps a book. The book measures 5 x 15 x 20 cm. Afterwards, she wants to tie a ribbon around the present. How much ribbon does Sina need? (adapted from Maaß, 2009)
O-items	Toothpaste	How many centimeters of toothpaste are used in one month? (adapted from Bülow et al., 2013)
	Birthday	Max celebrates his birthday. He wants to eat chocolate marshmallows with his guests. How many packs does he have to buy with his mother? (adapted from Maaß, 2009)

Table 2

Percentages of Realistic Responses (and Absolute Numbers in Parentheses)

Problem type	Item	Experimental group	Control group
P-items	Rope	3.9% (2)	8.9% (5)
	Present	3.9% (2)	1.8% (1)
O-items	Toothpaste	65.4% (34)	35.7% (20)
	Birthday	59.6% (31)	14.3% (8)

Table 3

Summary of the Logistic Regression Analysis

Items		β	*SE*	Wald	*p*	e^{β}	R^2
Comparisons between items							
Toothpaste-Rope		1.74	0.55	10.12	<.001*	5.67	.16
Toothpaste-Present		3.42	1.05	10.67	<.001*	30.56	.32
Birthday-Rope		0.53	0.61	0.77	.190	1.70	.01
Birthday-Present		2.22	1.08	4.22	.020	9.17	.14
Comparisons between groups							
P-items	Rope	-0.92	0.86	1.14	.144	0.40	.03
	Present	0.77	1.24	0.38	.268	2.16	.02
O-items	Toothpaste	1.17	0.40	8.56	.002*	3.22	.11
	Birthday	2.14	0.47	20.39	<.001*	8.46	.28

Table 4

Percentages of Realistic Responses (and Absolute Numbers in Parentheses)

Type	Item	First RS	Second RS	At least one RS	Both RS	Both UR	First RS second UR	First UR second RS
P-items	Rope	5.0 (3)	26.7 (16)	30.0 (18)	1.7 (1)	70.0 (42)	3.3 (2)	25.0 (15)
	Present	5.0 (3)	5.0 (3)	8.3 (5)	1.7 (1)	91.7 (55)	3.3 (2)	3.3 (2)
O-items	Tooth	76.7 (46)	75.0 (45)	78.3 (50)	73.3 (44)	21.7 (13)	3.3 (2)	1.7 (1)
	Birthday	63.3 (38)	60.0 (36)	65.0 (39)	58.3 (35)	35.0 (21)	5.0 (3)	1.7 (1)

Note. RS = Realistic solution; UR = Unrealistic solution.